OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4730

Mechanics 3

Tuesday **10 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

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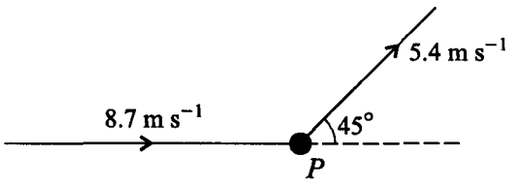
1



A particle P of mass 0.4 kg moving in a straight line has speed 8.7 m s^{-1}. An impulse applied to P deflects it through 45° and reduces its speed to 5.4 m s^{-1} (see diagram). Calculate the magnitude and direction of the impulse exerted on P. [7]

2 O is a fixed point on a horizontal straight line. A particle P of mass 0.5 kg is released from rest at O. At time t seconds after release the only force acting on P has magnitude $(1 + kt^2)$ N and acts horizontally and away from O along the line, where k is a positive constant.

(i) Find the speed of P in terms of k and t. [3]

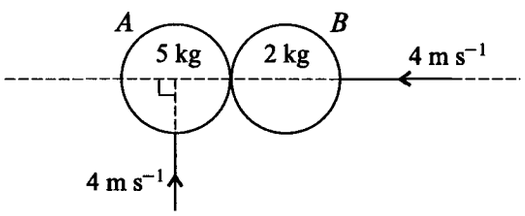
(ii) Given that P is 2 m from O when $t = 1$, find the value of k and the time taken by P to travel 20 m from O. [5]

3 A light elastic string has natural length 3 m. One end is attached to a fixed point O and the other end is attached to a particle of mass 1.6 kg. The particle is released from rest in a position 5 m vertically below O. Air resistance may be neglected.

(i) Given that in the subsequent motion the particle just reaches O, show that the modulus of elasticity of the string is 117.6 N. [4]

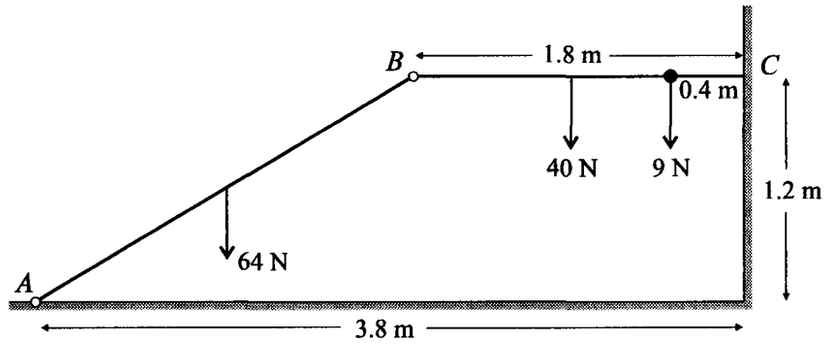
(ii) Calculate the speed of the particle when it is 4.5 m below O. [4]

4



Two uniform smooth spheres A and B, of equal radius, have masses 5 kg and 2 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision, A has speed 4 m s^{-1} and is moving perpendicular to the line of centres, and B has speed 4 m s^{-1} along the line of centres (see diagram). The coefficient of restitution is 0.75. Find the speed and direction of motion of each sphere immediately after the collision. [10]

5



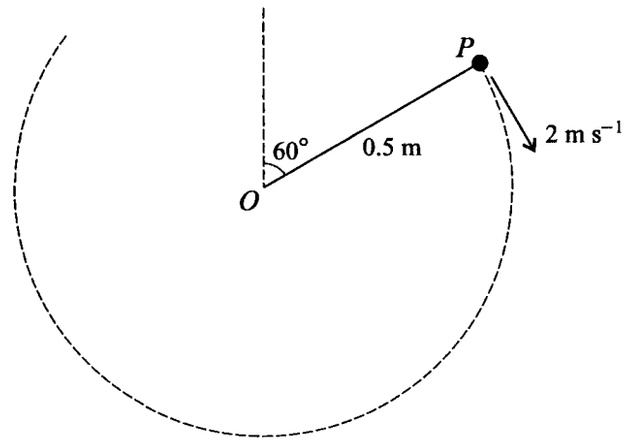
Two uniform rods AB and BC have weights 64 N and 40 N respectively. The rods are freely jointed to each other at B. The rod AB is freely jointed to a fixed point on horizontal ground at A and the rod BC rests against a vertical wall at C. The rod BC is 1.8 m long and is horizontal. A particle of weight 9 N is attached to the rod BC at the point 0.4 m from C. The point A is 1.2 m below the level of BC and 3.8 m from the wall (see diagram). The system is in equilibrium.

(i) Show that the magnitude of the frictional force at C is 27 N. [4]

(ii) Calculate the horizontal and vertical components of the force exerted on AB at B. [5]

(iii) Given that friction is limiting at C, find the coefficient of friction between the rod BC and the wall. [2]

6



One end of a light inextensible string of length 0.5 m is attached to a fixed point O. A particle P of mass 0.3 kg is attached to the other end of the string. With the string taut and at an angle of 60° to the upward vertical, P is projected with speed $2\,\text{m s}^{-1}$ (see diagram). P begins to move without air resistance in a vertical circle with centre O. When the string makes an angle θ with the upward vertical, the speed of P is $v\,\text{m s}^{-1}$.

(i) Show that $v^2 = 8.9 - 9.8\cos\theta$. [4]

(ii) Find the tension in the string in terms of θ. [4]

(iii) P does not move in a complete circle. Calculate the angle through which OP turns before P leaves the circular path. [4]

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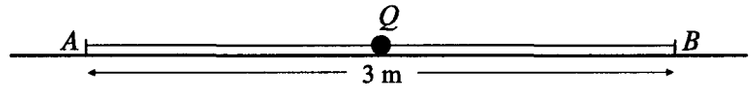
7



As shown in the diagram, A and B are fixed points on a smooth horizontal table, where $AB = 3$ m. A particle Q of mass 1.2 kg is attached to A by a light elastic string of natural length 1 m and modulus of elasticity 180 N. Q is attached to B by a light elastic string of natural length 1.2 m and modulus of elasticity 360 N.

(i) Verify that when Q is in equilibrium $BQ = 1.5$ m. [4]

Q is projected towards B from the equilibrium position with speed u m s^{-1}. Subsequently Q oscillates with simple harmonic motion.

(ii) Show that the period of the motion is 0.314 s approximately. [5]

(iii) Show that $u \leqslant 6$. [4]

(iv) Given that $u = 6$, find the time taken for Q to move from the equilibrium position to a position 1.3 m from A for the first time. [3]

1	$\pm(5.4\cos45° - 8.7)$	M1		For attempting to find Δv in **i** dir'n
		M1		For using I = m(Δ v) in **i** direction
	$I\cos\theta = \pm 0.4(5.4\cos45° - 8.7)$	A1		(= $\mp$ 1.953)
	$I\sin\theta = 0.4x5.4\sin45$	B1		(= 1.527)
	$I = \sqrt{(1.527^2 + 1.953^2)}$ or $\theta = \tan^{-1}[1.527/(-1.953)]$	M1		For using Pythagoras or trig.
	Magnitude is 2.48 $kgms^{-1}$	A1		
	Direction is 142° to original dir'n.	A1	[7]	Accept θ = 38.0° with θ shown appropriately
OR		M1		For using Impulse = mass x Δv
		M1		For appropriate use of cosine rule
	$I = 0.4\,(5.4^2 + 8.7^2 -$			
		A1		
	$2x5.4x8.7\cos45°)^{½}$			
	Magnitude is 2.48 $kgms^{-1}$	A1		
		M1		For appropriate use of sine rule
	$\sin\theta/5.4 = \sin45°/6.1976$	A1		
	θ = 38.0°	A1		

2	(i)	M1		For correct use of Newton's 2nd law
	$0.5dv/dt = 1 + kt^2$	A1		
	$v = 2t + 2kt^3/3$	A1	[3]	
				SR(max 1/3) for omission of mass but otherwise correct $v = t + kt^3/3$ B1
	(ii) $x = t^2 + kt^4/6$	M1		For integration w.r.t. t
	$2 = 1 + k/6$	M1		For substitution and attempting to solve for k
	$k = 6$	A1		
		M1		For attempting to solve quadratic in t^2 for t
	$t = 2$	A1	[5]	With no extra solutions

3	(i)	M1		For use of EE formula
	$EE = \lambda x (5-3)^2/(2 x 3)$	A1		
	$2\lambda/3 = 1.6 x 9.8 x 5$	M1		For equating EE and PE
	λ = 117.6 N	A1	[4]	AG
	(ii)	M1		For use of conservation of energy
	$0.5x1.6v^2 = 1.6x9.8x4.5 - 117.6x1.5^2/(2x3)$	A2,1,0		-1 each error
	v = 5.75 ms^{-1}	A1	[4]	

4	Perp. vel. of A after impact = 4	B1		
		M1		For using cons'n of m'm'tum // l.o.c
	[5x0] - 2x4 = 5a + 2b	A1		
		M1		Using N.E.L. // l.o.c.
	0.75 x 4 = b-a	A1		
		M1		For solving sim. equ.
	Speed of B is 1ms^{-1}; direction //l.o.c. and to the right	A1		
	$v_A=\sqrt{(4^2+(-2)^2)}$	M1		For method of finding the speed of A
	tan(angle) = 4/2	M1		For method of finding the direction of A
	Speed of A is 4.47 ms^{-1}; direction is 63.4° to l.o.c. and to the left	A1	[10]	
5	(i)	M1		For any moment equ. that includes F and all other relevant forces
	1.8F = 0.9x40 +1.4x9	A2,1,0		-1 each error
	Magnitude is 27 N	A1	[4]	AG
	(ii) Vertical comp. is 22 N downwards	B1		
		M1		For any moment equ. that includes X and all other relevant forces
	1.2X = (40+9-27)x(3.8-1.8) + 64 x1 (1.2X = 44 + 64)	A2,1,0 ft		-1 each error. ft wrong vert. comp.
	Horizontal comp. is 90 N to the left	A1	[5]	
	(iii) μ = 27/[90]	M1		For use of μ =F/R
	Coefficient of friction is 0.3	A1 ft	[2]	ft wrong answer in (ii)
6	(i)	M1		For use of conservation of energy
	$0.5x0.3v^2 - 0.5x0.3x2^2 = 0.3x9.8x0.5\cos 60 - 0.3x9.8x0.5\cos\theta$	A2,1,0		-1 each error
	$v^2 = 8.9 - 9.8\cos\theta$	A1	[4]	AG
	(ii)	M1		For using Newton's 2nd law radially
	$T + 0.3x9.8\cos\theta = 0.3v^2/0.5$	A1		
	$T + 2.94\cos\theta = 0.6(8.9 - 9.8\cos\theta)$	M1		For correct substitution for v^2
	Tension is$(5.34 - 8.82\cos\theta)$N	A1	[4]	Accept any correct form
	(iii)	M1		For using T = 0
	Basic value $\theta = 52.7°$	A1 ft		ft any T of the form a - bcosθ
	Angle = (360-52.7) - 60	M1		
	Angle turned through is 247°	A1	[4]	

7	(i)	M1		For using T = λ e/L once
	For 180e/1 or 360(0.8-e)/1.2 **or** T_A = 180 x 0.5/1 or T_B = 360 x 0.3/1.2	A1		
	480e = 240 or T_A = 90, T_B = 90	M1		For using $T_A(e) = T_B(e)$ or attempting to show $T_A = T_B$ when BQ = 1.5
	BQ = 1 + 0.5 = 1.5 m or $T_A = T_B$	A1	[4]	AG
	(ii) $T_B = 360(0.3 - x)/1.2$	B1		
	$T_A = 180(0.5 + x)$	B1		
	$1.2d^2x/dt^2$ = 300(0.3-x) - 180(0.5+x)	M1		For using Newton's 2nd law
	$d^2x/dt^2 = -400x$	A1		
	Period is $2\pi/\sqrt{[400]}$ = 0.314 s	A1	[5]	AG
	(iii)	M1		For using $T_B = 0$
	Max amplitude = 1.5 - 1.2 = 0.3 m	A1		
	amplitude = $u/\sqrt{400}$ or $180x0.5^2/(2x1) + 360x0.3^2/(2x1.2) + \frac{1}{2}\,1.2u_{max}^2 = 180x0.8^2/(2x1)$	M1		For using Amp. = u/ω or 'energy at equil. pos'n = energy at max. displ.'
	Maximum value of u is 6	A1	[4]	AG
	(iv) $-0.2 = 0.3\sin 20t$	M1		For relevant trig. equation
	$20t = 0.7297 + 3.142$	M1		For method of obtaining relevant solution
	Time taken is 0.194s	A1	[3]	